

Semester 1 Final Exam Review

1) $y = 2$
 $y = -2$

2) $k = -7$

3) $\frac{dy}{dx} = x - 2\cos y \frac{dy}{dx}$

$\frac{dy}{dx} = 2 - 2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2}{3}$

4) $\frac{1}{x} - \frac{y'}{y} = y'$

$\frac{1}{4} - \frac{1}{4}y' = y'$

$\boxed{y' = \frac{1}{5}}$

5) $\frac{1}{\sqrt{1-x^2}} = \frac{y'}{y}$

$y' = \frac{y}{\sqrt{1-x^2}}$

6) $g'(3) = \frac{1}{f'(g(3))}$

$= \frac{1}{f'(1)} = \frac{1}{4}$

7) $(h^{-1})'(2) = \frac{1}{h'(h^{-1}(2))}$

$= \frac{1}{h'(1)}$

$= \frac{1}{8}$

8) $g'(1) = \frac{1}{f'(g(1))}$

$\sin x + 2x + 1 = 1$

$\sin x + 2x = 0$

$x = 0$

$f'(x) = \cos x + 2$

$f'(0) = 3$

$\boxed{g'(1) = \frac{1}{3}}$

9) $2y \frac{dy}{dx} - 2x^2 \frac{dy}{dx} - 4xy = 0$

$\frac{dy}{dx} = \frac{4xy}{2y - 2x^2} = \frac{2xy}{y - x^2}$

10) $\frac{1}{\sqrt{1-a^2}} = 2$

$\frac{1}{1-a^2} = 4$

$1 = 4 - 4a^2$

$\frac{3}{4} = a^2$

$a = \frac{\sqrt{3}}{2}$

11) $y - 4 = 3(x - 1)$

$L(1.2) = 4 + 3(0.2)$

$f(1.2) \approx 4.6$

12) $(4, 5)$

$$13) (0, 1.256) \\ (2.056, \infty)$$

$$14) \frac{dA}{dt} = \frac{dC}{dt} \\ 2\pi r \frac{dr}{dt} = 2\pi \frac{dr}{dt} \\ \boxed{r = 1}$$

$$15) V = \frac{10}{3}\pi r^2 - \frac{1}{3}\pi r^3 \\ \frac{dV}{dt} = \frac{20}{3}\pi r \frac{dr}{dt} - \pi r^2 \frac{dr}{dt} \\ \frac{dV}{dt} = \frac{120\pi}{3}r - 6\pi r^2$$

$$16) 4.298 \text{ mpg} \\ \text{hour}$$

$$17) V = \frac{4}{3}\pi r^3 \quad A = \pi r^2 \\ \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \\ 2 = 2\pi(4) \frac{dr}{dt} \\ \frac{\pi}{4} = \frac{dr}{dt} \\ \boxed{\frac{dV}{dt} = 16\pi^2 \text{ cm}^3/\text{sec}}$$

$$18) 1$$

$$19) 2$$

$$20) -2$$

$$21) g'(x) = 12x^2 + 6x - 6 = 0 \\ 2x^2 + x - 1 = 0 \\ (2x-1)(x+1) = 0 \\ x = \frac{1}{2} \quad x = -1$$

x	g(x)
-2	-7
-1	6
$\frac{1}{2}$	$-\frac{3}{4}$
1	2

$$22) f(x) = \frac{\ln x}{x} \quad D: (0, \infty) \\ f'(x) = \frac{x(\frac{1}{x}) - \ln x}{x^2} = \frac{1 - \ln x}{x^2} \\ x = e$$

$$\boxed{f(e) = \frac{1}{e}}$$

$$23) f'(x) = 3x^2 + 5 \quad \frac{f(3) - f(1)}{3-1} = \frac{42-6}{2}$$

$$24) \text{ II}$$

$$3x^2 + 5 = 18$$

$$3x^2 = 13$$

$$\boxed{x = \sqrt{\frac{13}{3}}}$$

$$25) f'(x) = 6x^{-1/3} + 3$$

C.V.

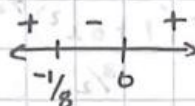
$$x = 0$$

$$6x^{-1/3} = -3$$

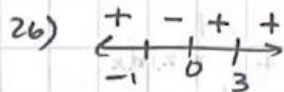
$$x^{-1/3} = -2$$

$$\frac{1}{x^{1/3}} = -2$$

$$x = -\frac{1}{8}$$



$\boxed{\text{min @ } x = 0}$



$\boxed{\text{max @ } x = -1}$

27) X

$$28) y' = 15x^4 + 40x^3$$

$$y'' = 60x^3 + 120x^2 = 0$$

$$60x^2(x+2) = 0$$

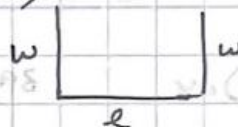
$$x = 0 \quad x = -2$$

$\boxed{\text{p.o.} \pm \text{ @ } x = -2}$

$$29) h''(x) = 2f''(x)$$

Two P.O.F

30)



$$18 = 2w + l \rightarrow l = 18 - 2w$$

$$A = l \cdot w$$

$$A = 18w - 2w^2$$

$$A' = 18 - 4w = 0$$

$$w = \frac{9}{2}$$

$$A\left(\frac{9}{2}\right) = 81 - \frac{81}{2} = \frac{81}{2} \text{ m}$$

$$31) \int (5x^{3/2} - 5x^3) dx$$

$$2x^{5/2} - \frac{5}{4}x^4 + C$$

$$32) f(x) = 2\ln x + C$$

$$5 = 2\ln \sqrt{e} + C$$

$$5 = 1 + C$$

$$4 = C$$

$$f(e) = 2\ln e + 4 = \boxed{6}$$

$$33) \int \frac{e^x}{1+e^x} dx$$

$$\int (1+e^x)^{-1} e^x dx$$

$$\boxed{\ln(1+e^x) + C}$$

$$34) u(-1) = -2 \quad du = 2x dx$$

$$u(4) = 13 \quad \frac{1}{2} du = x dx$$

$$\frac{1}{2} \int_{-2}^{13} u^5 du \quad \textcircled{C}$$

$$35) \int_0^4 f(x) dx = \int_0^1 f(x) dx + \int_1^4 f(x) dx$$

$$-3 = 2 + \int_1^4 f(x) dx$$

$$-5 = \int_1^4 f(x) dx$$

$$\int_1^4 (3f(x) + 2) dx$$

$$3(-4) + 6 = \boxed{-6}$$

$$36) \int_0^1 x(1+8x^2)^{1/2} dx$$

$$\frac{1}{16} \frac{(1+8x^2)^{3/2}}{3/2} \Big|_0^1$$

$$\frac{1}{24}(27) - \frac{1}{24} = \boxed{\frac{13}{12}}$$

$$37) \int f(x) \sin x dx = -f(x) \cos x + \int 4x^3 \cos x dx$$

$$u = f(x) \quad v = -\cos x$$

$$du = 4x^3 \quad dv = \sin x dx$$

$$\boxed{f(x) = x^4}$$

$$38) \int_0^4 f(x) dx = 8 \quad \int_0^4 x f'(x) dx$$

$$u = x \quad v = f(x)$$

$$du = dx \quad dv = f'(x) dx$$

$$x f(x) \Big|_0^4 - \int_0^4 f(x) dx$$

$$4f(4) - 0 - 8$$

$$\boxed{-20}$$

$$39) \int_2^4 f(x) g''(x) dx$$

$$u = f(x) \quad v = g'(x)$$

$$du = f'(x) dx \quad dv = g''(x) dx$$

$$f(x) g'(x) \Big|_2^4 - \int_2^4 f'(x) g(x) dx$$

$$f(4)g'(4) - f(2)g'(2) - \int_2^4 f'(x)g(x) dx$$

$$13(7) - 7(1) - [27 - 16]$$

$$84 - 21$$

$$\boxed{73}$$

$$40) \int_0^1 \frac{5x+8}{x^2+3x+2} dx$$

$$\frac{A}{x+2} + \frac{B}{x+1} = \frac{5x+8}{x^2+3x+2}$$

$$A(x+1) + B(x+2) = 5x+8$$

$$\frac{x=-1}{B=3} \quad \frac{x=-2}{A=2}$$

$$\int_0^1 \left[\frac{2}{x+2} + \frac{3}{x+1} \right] dx$$

$$2 \ln(x+2) + 3 \ln(x+1) \Big|_0^1$$

$$2 \ln 3 + 3 \ln 2 - (2 \ln 2)$$

$$2 \ln 3 - \ln 2 = \boxed{\ln \frac{9}{2}}$$

$$41) \int_1^\infty \frac{x^2}{(x^3+2)^2} dx$$

$$\lim_{b \rightarrow \infty} \int_1^b x^2 (x^3+2)^{-2} dx$$

$$\lim_{b \rightarrow \infty} \left[\frac{1}{3} (x^3+2)^{-1} \Big|_1^b \right] = \lim_{b \rightarrow \infty} \left[-\frac{1}{3(x^3+2)} \right]$$

$$0 + \frac{1}{9}$$

$$\boxed{\frac{1}{9}}$$

$$42) \int \frac{x^3 + 5}{x^2} dx$$

$$\int [x + 5x^{-2}] dx$$

$$\boxed{\frac{1}{2}x^2 - 5x^{-1} + C}$$

$$43) T_3 = \frac{1}{2}(2) [4 + 2k + 16 + 12] = 52$$

$$2k + 32 = 52$$

$$\boxed{k = 10}$$

$$44) \frac{1}{4} \int_0^4 R(t) dt = 14.691$$



$$\textcircled{B} \quad L < T < J < R$$

$$46) v(t) = \sin(2t)$$

$$x(0) = 4 + \int_{\pi/2}^0 \sin(2t) dt$$

$$= 4 + \left[-\frac{1}{2} \cos(2t) \right]_{\pi/2}^0$$

$$= 4 + \left[-\frac{1}{2} + 0 \right]$$

$$= \boxed{\frac{7}{2}}$$

$$47) 200 + \int_0^4 R(t) dt = 175^\circ$$

$$48) 1.582$$

$$49) 1.029$$

$$50) 80.114$$

$$51) \textcircled{D}$$

$$52) L = \int_1^7 \sqrt{1 + (f'(x))^2} dx$$

$$2 \left[(1 + f'(1)^2) + (1 + f'(3)^2) + (1 + f'(5)^2) \right]$$

$$2 \left[(1 + 4) + (1 + 1) + (1 + 0) \right] = \boxed{16}$$